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MULTI-OBJECTIVE APPROACH FOR ROBUST DESIGN OPTIMIZATION PROBLEMS

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ABSTRACT

The paper demonstrates the main capabilities of IOSO (Indirect Optimization based on Self-Organization) technology algorithms, tools and software, which can be used for the optimization of complex systems and objects. IOSO algorithms have higher efficiency, provide a wider range of capabilities, and are practically insensitive with respect to the types of objective function and constraints. They could be smooth, non-differentiable, and stochastic, with multiple optima, with the portions of the design space where objective function and constraints could not be evaluated at all, with the objective function and constraints dependent on mixed variables, etc. The capabilities of IOSO software are demonstrated using examples of solving complex multi-objective (up to 8 simultaneous objectives) problems, which are solved in deterministic and robust design optimization statements. The results of this paper show the Pareto set probability statement, which decreases technical risks when developing modern objects and systems with the highest level of efficiency.

INTRODUCTION

Designing a complex technical system in present-day conditions is impossible without the use of optimization techniques. In fact, design and optimization processes do represent a single whole. While designing a technical system and picking up its parameters the designer had always been implicitly assessing possibilities of practical implementation of the system.

The rise of the complexity of systems as well as the number of parameters needed to be coordinated with each other in an optimal way have led to the necessity of using mathematical modeling of systems and application of optimization techniques. In this situation the designer focuses on working out of an adequate mathematical model and the analysis of the results obtained. Choosing optimal parameters for the system being designed is done through the use of formal mathematical optimization procedures. The use of such an approach exempts the designer of routine work aimed to select optimal combinations of variable parameters, allowing him to set and solve extremely complex problems of optimal designing. However, solutions obtained by means of mathematical modeling and optimization techniques in most cases are hard to implement in real life. This is largely due to the fact that while stating and solving optimization tasks by traditional (deterministic) approach, as a rule, various uncertainties influencing the efficiency of the designed system in real life conditions are not taken into consideration.

In recent years, probabilistic design analysis and optimization methods have been developed to account for uncertainty and randomness through stochastic simulation and probabilistic analysis. Totality of such methods can be treated as the new scientific direction, named "Robust Design Optimization" (RDO). The distinct feature of this direction is the use of probability criteria to evaluate the technical system quality.

Despite great variety of problem statements and the methods to solve optimization problems in conditions of uncertainty, there are a number of common problems that should be addressed by the investigators [4]. In this paper we are only indicate these problems:

- Identifying the main uncertainties, affecting the design (typically: uncertainties in variables real-life realization; uncertainties in environmental conditions; mathematical model accuracy).
- Selecting the probability criteria (for example: mean value of efficiency; efficiency value deviation; probability that efficiency value is no worse than the one given; efficiency value

ensured with probability no less than the one given).

- Selecting of procedure for probability criteria evaluation (analytical approach; Monte-Carlo technique; some special techniques, for example IOSO Technology technique [4]).
- Selecting the optimization method.

The problem of reasonable choice of the procedure for calculation of probabilistic criteria as well as the optimization technique becomes more complicated if RDO problem assumes multiple objectives. As mentioned above, RDO problems are multi-objectives ones by nature since them, in fact, assume compromises between what can be implemented in real life and the probability of achieving of obtained results. In these conditions the ultimate choice of the project for the investigated system is to be made based on the analysis of totality of Pareto-optimal projects, obtained with probabilistic and deterministic criteria. Within the framework of IOSO technology we have elaborated a number of multi-objectives algorithms to solve such complex problems. The main advantages of these algorithms over traditional mathematical programming approaches are the following [3]:

- convolution approaches are not used in solving multi-objective problems;
- the algorithms determine the desired number of Pareto-optimal solutions, so that these solutions are uniformly distributed in the space of objective functions values;
- it is possible to solve the optimization problems for the objective functions of complex topology: non-convex, non-differentiable, with many local optima;
- relatively small number of probability indexes evaluations;
- it is possible to naturally employ the parallelization of the computational process.

These advantages are the basis for the wide use of the proposed method in the real-life problems.

IOSO ALGORITHMS ESSENCE

In general the multi-objective optimization problem consists in minimization of a vector of n objective functions

$$min(f_i(x,e)) \quad for \quad i=\overline{1,n}, \tag{1}$$

subject to a vector of inequality constraints:

$$g_j(x,e)\leq 0 \quad for \quad j=\overline{1,m}, \tag{2}$$

and a vector of equality constraints:

$$h_q(x,e)=0 \quad for \quad q=\overline{1,k}, \tag{3}$$

Here x – is a vector of variables; e – vector of environmental conditions.

Our approach is based on the widespread application of the response surface technique, which depends upon the original approximation concept, within the frameworks which adaptively use global and middle-range multi-point approximation. One of the advantages of the proposed approach is the possibility of ensuring good approximating capabilities using the minimum amount of available information. This possibility is based on self-organization and evolutionary modeling concepts. During the approximation, the response surface function structure is being evolutionarily changed, so that it allows for the successful approximation of the optimized functions and constraints having sufficiently complicated topology.

Every iteration of IOSO consists of two steps. The first step is the creation of an analytical approximation of the objective function(s). The second step is the optimization of this approximation function. The optimization of the response function is performed only within the current search area during each iteration of IOSO.

This step is followed by a direct call to the mathematical analysis model or an actual experimental evaluation for the obtained point. During the IOSO operation, the information concerning the behavior of the objective function in the vicinity of the extremum is stored, and the response function is made more accurate only for this search area. While proceeding from one iteration to the next, the following steps are carried out: modification of the experiment plan; adaptive selection of the current extremum search area; choice of the response function type (global or middle-range); transformation of the response function; modification of both parameters and structure of the optimization algorithms; and, if necessary, selection of new promising points within the researched area.

When solving RDO we have the same formal statement (1)…(3).

The attempt to include uncertainties while robust design problem formalization results in the necessity to consider relations:

$$x=x(\bar{x},x_x); \tag{4}$$

$$e=e(\bar{e},x_e); \tag{5}$$

$$f(x,e)=\Psi(\bar{f}(x,e),x_f(x,e)), \tag{6}$$

where $\bar{x},\bar{e},\bar{f}$ are ideal vectors of variable parameters, environmental conditions and the ideal mathematical model; $x=(x_x,x_e,x_f)$ is the vector of random values including uncertainties in implementation of variable parameters, environment conditions and the mathematical model accuracy. Generally, to solve RDO problem one must be able to determine the system efficiency values $y=f(x,e)$ for given values of $\bar{x},\bar{e}$, and hence to know the laws of distribution of x vector components and functional dependence of $\Psi(\bar{f},x_f)$. It means that for RDO we must define some probability criteria (objectives and constraints) for each iteration. Robust design optimization problems (even for a single chosen efficiency) are in essence the multi-objective ones and appropriate techniques to solve them should be used.

The main problem occurring while solving RDO problems is determining probabilistic criteria values. There are various approaches to solving this problem [1, 2, 4, 6, 7].

When solving real-life RDO problems we use basic algorithms of IOSO technology with some modifications. The effective noise-proof feature of these algorithms enables us to evaluate probability criteria by means of the Monte-Carlo technique with an extremely small amount of statistical tests at each search iteration.

IOSO SOFTWARE FEATURES

IOSO Technology implements the new evolutionary response surface methodology. These algorithms are practically insensitive with respect to the types of objective function and constraints: smooth, non-differentiable, stochastic, with multiple optima, with the portions of the design space where objective function and constraints could not be evaluated at all, with the objective function and constraints dependent on mixed variables, etc. [3, 5].

A comparison of IOSO Technology algorithms with various up-to-date nonlinear optimization methods has been made. For comparison, we chose well-known test functions, which were complex, nonlinear problems of conditional and unconditional optimization (total 30 test optimization problems proposed by E. Sandgren [8]). When comparing optimization methods, we considered one complex criterion. This criterion evaluates the efficiency of optimization strategy taking into account the dimensionality of the problem, the number and type of constraints (equality or inequality), the accuracy of solution determination as well as the number of function evaluations required for obtaining the solution. The main results is that the IOSO basic algorithm can compete successfully with well-known optimization methods [5].

Software and tools of IOSO Technology consist of several independent algorithms. All IOSO technology algorithms were developed according to the single concept of formulating optimization problems, providing initial data, data exchange with the user's program, and analysis of the obtained results.

IOSO Technology Tools implement highly efficient evolutionary self-organizing algorithms. The efficiency is guaranteed by internal adaptive choice of the algorithm suitable for each particular problem. This feature results in solving complex optimization problems with a minimal number of evaluations of the system mathematical model [2…5].

This optimization procedure is universal. It is uniquely powerful according to the relationship between the required number of calls to the analysis module and response topography complexity. On smooth object function it works as well as gradient methods. However, for complex (more probable to be faced by a designer in practice) object functions, having incomputability areas, discontinuities, multiple extremums and noise, the number of function calls required to find the global extremum is being increased considerably, while gradient methods are inapplicable for such task solutions.

For example, Figure 1 shows the results of optimization of well know Levy #9 test problem with 4 design variables [10]. This optimization problem is a multi-extremum optimization function with more then 626 local minima.

Figure 2 illustrates the results for the same problem which has the following modification:

$$y_{nondiff}=\begin{cases} y+10^{6-i}, if \ \ y\ >10^{5-i}, i=\overline{1,17} \\ y \end{cases} \qquad (7)$$

This means that this test function is discontinuous. It has 17 levels of decreasing shock patterns.

IOSO tools and software work with only executable modules written to represent mathematical models. This significantly facilitates the customizing of the interaction of user's model and the optimization procedure since it does not require either shared PC memory spaces for data exchange or specific programming language to write the analysis code. Data exchange is provided by means of text files on a disk drive, making it easy to integrate the analysis codes into IOSO tools and software package. IOSO software has user friendly GUI and is simple to use. The parameters of IOSO technology are pre-programmed and are adaptively changing during the search for extremum without the user's intervention. Most of the algorithm's tunings are done internally, that is, they are hidden from the user who is not required to have any knowledge of nonlinear programming or optimization procedures. The only important thing for the user to understand is the physics of the problem and to have a mathematical model of the system. Creating an interface between IOSO and mathematical model typically takes several minutes.

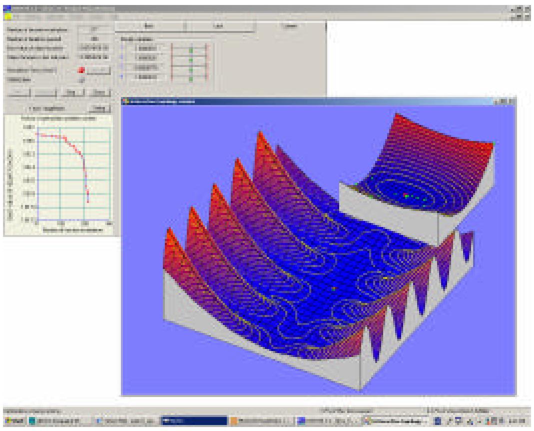


Figure 1. Optimization of Levy #9 test problem.

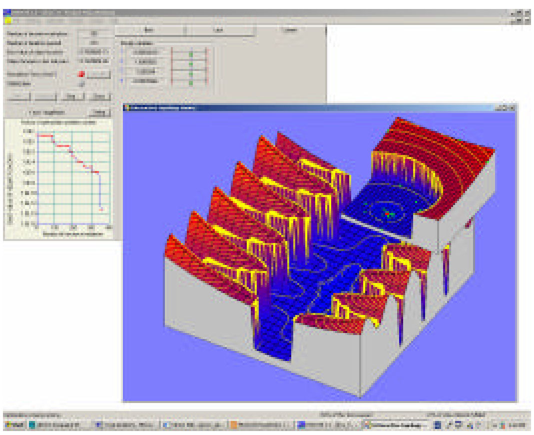


Figure 2. Optimization of Levy #9 test problem with discontinuous.

The optimization process is visually represented in real time (displayed in current values of the design variables and their bounds) representing the objective function history. The user is able to control the optimization process. Users can interrupt the optimization process to tune up parameters with the ability to restart from the specified point, thus, cleaning up a "hanged" or crashed user's mathematical model.

MULTIDISCIPLINARY OPTIMIZATION OF AIR-ENGINE

The purpose is to obtain the totality of Pareto-optimum combinations of air engine and aircraft parameters. This means that we must use a mathematical model of air engine and aircraft that allow defining objectives and constraints for different design variables of air engine and aircraft. We used analysis codes of air engines and aircraft for this research which was developed earlier. The analysis code of air engines allows one to define the performance characteristics of the engine for given parameters of engine operation process. It means that we can calculate specific fuel consumption and thrust, with external resistance included, for any flight operating modes of aircraft; weight, size engine's parameters; engine's life period; level of engine noise; design, operating mode and maintenance costs of the engine for the current value of the operation process engine parameters.

Performance characteristics of an aircraft are calculated by using a mathematical model developed by the Central Institute of Aircraft Motors (CIAM) [9]. This model allows one to define the main objectives of subsonic and supersonic aircraft for given design parameters and performance characteristics of an engine, and the geometry of aircraft, at the various variants of flight conditions and different operating modes of aircraft. For example, we can calculate passenger-by-kilometer fuel consumption, direct maintenance expenditures, maintenance costs, terrain noise level, take-off runway, maximum of altitude, maximum Mach number for different parameters of the operation process of the engine, and the different geometry of aircraft.

As shown in the preliminary analysis, while solving design problems (one for aircraft and engines, for example) incomputable areas of values of objective function and constraints may exist. This can be conditioned by both the impossibility of project existence at a certain combination of design variables, and the instability of numerical schemes used as mathematical models. This can even lead to the crash of the user's application.

Generally, it is a multi-objective constrained non-linear optimization problem with a region that can use crash analysis codes. We used IOSO NM 1.0 software for this research, which allowed the solution of this type of optimization problems [10]. We tried to find the best design (Pareto set), including, first, multi-point operating modes of aircraft and air engine, second, different flight programs of aircraft (really we use 5 different flight programs according to the requirements of designers). This means that we have no design operating mode for aircraft and air engine. We must improve some integral objectives, which describe the efficiency of this complex object including each operating mode (a set of different operating modes of flights).

Deterministic optimization

Purpose: to obtain the totality of Pareto-optimum combinations of air engine and aircraft geometry parameters for regional subsonic jet.

Problem features:

Design variables: - total compressor pressure ratio; low pressure compressor (fan) pressure ratio; bypass ratio; temperature before turbine, parameters of control system, and geometry parameters of aircraft (total 10 design variables).

Objectives: the main efficiency indexes of aircraft (passenger-by-kilometer fuel consumption, direct maintenance expenditures, terrain noise level, take-off runway etc. (total 8 objectives).

Constraints: design requirements of aircraft, maximum temperature before turbine, maximum pressure in exit of compressor, stall margins of both compressor at all operating modes, etc (total 26 constraints).

As example, Figure 3 shows Pareto set for two obectives. It can be seen that most of points of Pareto set allow a higher level of efficiency then is required (1.0 is required level). Note that each point of the Pareto set corresponds to a different operating mode for engine parameters and the geometry parameters of the aircraft. We think that such compromize presentation may be used for choosing the best technical solution.

Analysis of more then two objectives is not so evident. For example, Figure 4 show distribution of objectives for some different points of the Pareto set. Maybe point # 8 is the best because in this case we can improve all objectives by more then 2 %.

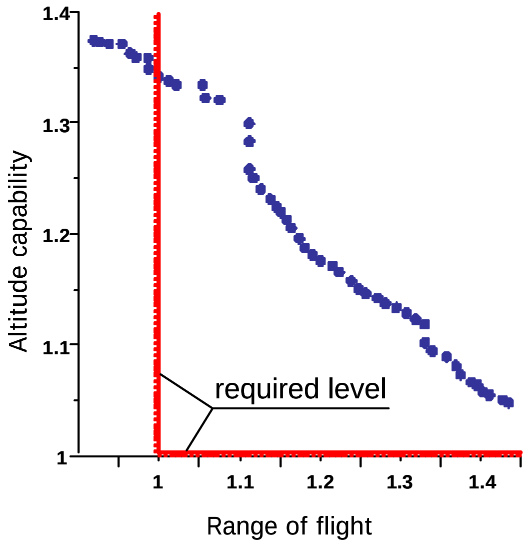
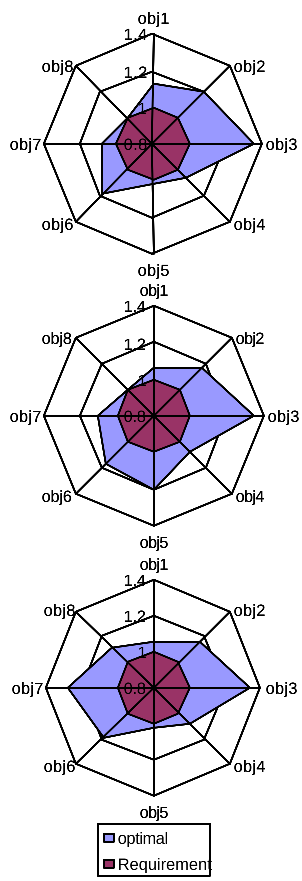


Figure 3. Pareto set.



1st point of the Pareto-set

6st point of the Pareto-set

8st point of the Pareto-set

Figure 4. Selected points of the Pareto set.

Robust design optimization

We used deterministic statements for the last problem. It means that we have no information about the probability of the realization of these results. That is why we cannot be sure that this efficiency can be realized in practice. It is very important to minimize the risk of realization failure to develop a modern complex system and objects. It is well-known that we can use a RDO approach for the solution of these problems [1, 2, 4, 6, 7]. It means that we must use probabilistic objectives for this optimization problem and calculate these criteria by each interaction. We used distributions of design variables for this RDO solution, which was based on many years of aircraft and air engine development. As shown the analysis of each design parameter has a different distribution. We have approximated this experimental data and used it for numerical research.

In this research we chose the probability of an objective as the stochastic criteria, because this type of stochastic criteria can guarantee high quality determination solution for Robust Design Optimization [4]. For this research case we have multi-objective constrained optimization problems with a crash analysis code. For this research we used IOSO RM 1.0 software [11].

Purpose*:* to research the possibilities of design requirements for commercial supersonic aircraft ensuring (the use of probability objectives).

Problem features:

Design variables: - total compressor pressure ratio; low pressure compressor (fan) pressure ratio; bypass ratio; temperature before turbine, parameters of control system, and geometry parameters of aircraft (total 10 design variables).

Objectives: the probabilities of main efficiency indexes of aircraft (passenger-by-kilometer fuel consumption, direct maintenance expenditures, terrain noise level, take-off runway etc. (total 8 objectives).

Constraints: many design requirements of aircraft, maximum temperature before turbine, maximum pressure in exit of compressor, stall margins of both compressors at the all operating modes, etc. (total 26 constraints).

The first part of this problem includes the solution of this problem in a deterministic statement. An example of a deterministic solution is Figure 3, which shows the Pareto set for flight and altitude flight capabilities. Then we test all these Pareto points in a probability statement using the preset distribution of each design variable. Then we defined the value of objectives for different levels of probability using numerical random research for a given distribution of each design variable. For this estimation we used 10000 numerical calls. The main results are shown in Figure 5 (square label, filled field is the level of the requirements of this project).

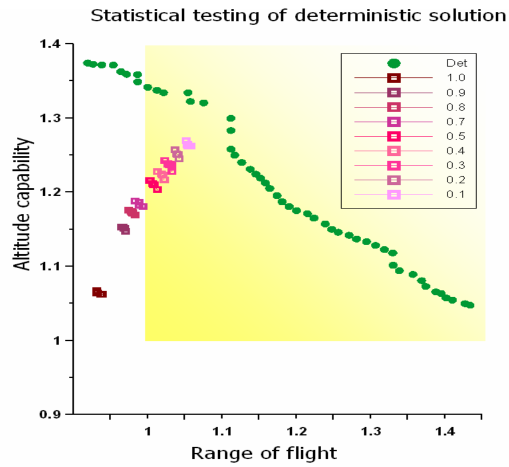


Figure 5. Pareto set for range of flight and altitude flight capabilities for deterministic statement.

First, one can see that the Pareto set in a probabilistic statement is very small. It means that we have low level of compromise between these objectives. As shown in analysis we have the same situation for other objectives. Second, both objectives are decreased. This means that one cannot ensure the level of efficiency, which was reached by a deterministic statement. Moreover, we cannot ensure the requirements of the project (1.0). This is impractical because the project cannot reach the required efficiency. In other words, a deterministic solution is a nice project, but we must understand that it is a project on paper only. We can never reach this level of efficiency if we try to realize this project for real-life objects. Note that each company has their own level of development and production. This means that we must develop a particular project using the specific uniqueness of this company, which has its own cycle of development and production for each object. In our case we use probabilistic properties for design variables only. In real-life we must use information about the accuracy of the analysis code, which we use for developing each object. For example, each company has variable levels of accuracy for their computer code analysis. The experimental research is also used for the development of modern objects with various levels of accuracy. Moreover we must have a clear understanding that we cannot consider all physical phenomena during the development of a project.

You can find many different statements and results for the solution of RDO problems in other papers [1, 2, 4, 6, 7]. Some of these problems can be solved if we use probabilistic statements. The general ideas of the RDO approach are explained next. We must find design variables, which allow us to ensure a high level of probability for the realization of a project. It means that we must find design variables where we have a low level of divergence of objectives for a given level of production (this includes all aspects which we discussed earlier). Note that the purpose of robust analysis and robust design optimization are very different. Robust analysis means that we study some current solutions near one point only. This point is the extrema of objectives. Robust design optimization means that we must find solutions (design variables), which ensure the best value for efficiency with maximum or given levels of probability for this object.

Figure 6 shows Pareto set for Robust Design Optimization. Comparative analysis (Fig.5 and Fig.6) is shown next. First, the RDO approach allows finding a solution, which can be realized with high-level probability. Second, we have compromises between these objectives. It means that we can choose a different solution from the Pareto set, which has different levels of efficiency for aircraft. Third, if one compares the results of a deterministic and RDO approach, we can ensure the highest-level of realizing the efficiency for aircraft for the same level probability. Fourth, we cannot ensure design requirements with probability P = 100%, but we can realize the project parameter with a probability P=98% (nearby P = 100%). Moreover, we have a Pareto set for this project with alternative variants which can be realized with probability P > 90%. This means that we have a level of freedom connected with choosing some alternative projects.

Figure 7 illustrates the main particularity and quality of Robust Design Optimization. For design requirements we can ensure the probability of realization P=98% for a range of flight conditions if we used the RDO procedure. For a deterministic statement we can only have P=63%. Thus, a deterministic approach cannot guarantee the needed level for range of flight, because probability P=63% is a lower level and guarantee of the requirement level for range of flight is an important contingency in statistical terms. Note that a deterministic solution for range of flight can be 0.95 only for the same level of probability P=98%. In other words, we cannot ensure the needed range of flight if we use a deterministic procedure. However, we can find it if we use the Robust Design Optimization approach. We have a different situation with altitude capabilities. Both approaches have approximately the same level of altitude capabilities for the same value of probability. Why? First, it is very easy to reach the level of this requirement. Second, all points of the Pareto set for the deterministic case allows us to ensure an increment of altitude capabilities more then 5% with reference to the requirement level.

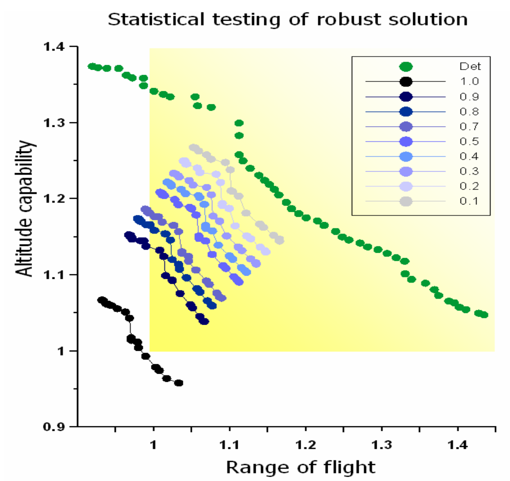


Figure 6. Pareto set for range of flight and altitude flight capabilities for Robust Design Optimization statement.

This means that we have some reserve of altitude capabilities, which can be used for the improvement of probability for a deterministic solution. Thus, the requirement for altitude capabilities is not critical for this project. We can ensure a higher level of this objective (altitude capabilities) without decreasing other objectives. Typically, probabilistic research and robust analysis decrease the efficiency of objects (for example, the decrease of flight range in this case). But, for altitude capabilities we have a large reserve of increment. It is enough to guarantee a high level of probability for the robust analysis stage. Thus, the results of RDO research give us very important information about an object. First, what is the possibility of improving each objective? Second, which objectives decrease the efficiency of the project? The last question is more important for practice. Can we exchange the formulas in this problem or must we find a new technical solution for this design (for examples, different configuration of aircraft, another schema of engine, including additional design variables etc). This information can help us to formulate this statement in a more correct form for future research.

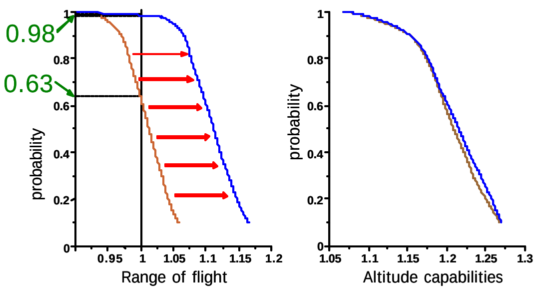


Figure 7. Change of flight range and altitude flight capabilities depending on the probability of realization.

CONCLUSIONS

A new robust optimization algorithm (IOSO) was shown to be a highly efficient and reliable tool for multi-objective optimization in deterministic and probabilistic statements. We tried to demonstrate that robust design optimizations ensure for a higher level of probability of realization for real-life technical solution.

A Pareto set in probabilistic statements allows for a decreased technical risk of development for new modern higher qualitative objects and systems. All of this research demonstrates some of the possibilities of IOSO tools and software.

The examples relate specifically to air engine and aircraft. However, this technology has been highly successful in use for many different areas and it can be used in a wide range of fields.

REFERENCES

1. S.M. Batill, J.,E. Renaud and X.-Y. Gu, “Modeling and Simulation Uncertainty in Multidisciplinary Design Optimization”, AIAA paper 2000-4803 (2000).

2. I.N. Egorov, “Optimization of a Multistage Axial Compressor. Stochastic Approach”, ASME paper 92-GT-163 (1992).

3. I.N. Egorov, and G.V. Kretinin, "Search for Compromise Solution of the Multistage Axial Compressor’s Stochastic Optimization Problem", World Publishing Corporation, Aerothermo-dynamics of internal flows III, Beijing, China, 1996, pp. 112-120.

4. I.N. Egorov, G.V. Kretinin, and I.A. Leshchenko, “How to Execute Robust Design Optimization”, AIAA paper 2002-4328 (2002).

5. I.N. Egorov, G.V. Kretinin, I.A. Leshchenko and S.V. Kuptzov, “IOSO Optimization Toolkit – Novel Software to Create Better Design”, AIAA paper 2002-4329 (2002).

6. P.N. Koch, B. Wujek, and O. Golovidov, “Parallel Implementation of Probabilistic Design Optimization in an MDO Framework”, AIAA paper 2000-4805 (2000).

7. J. Marczyk, “Stochastic Multidisciplinary Improvement: Beyond Optimization”, AIAA paper 2000-4929 (2000).

8. E. Sandgren, "The Utility of Nonlinear Programming Algorithms", Ph.D. thesis, Purdue University, IN, USA, (1977).

9. J.R. Yugov, and O.D. Selivanov, “Principles of Aircraft and Engine Integration”. “Mashinostroyenie”, Moscow, Russia (1989).

10. Users Guide, IOSO NM, version 1.0, IOSO Technology Center, Moscow, Russia, 2002.

11. Users Guide, IOSO NS Robust, version 1.0, IOSO Technology Center, Moscow, 2003.